(1) If ϕ is a differentiable scalar field.

Then ~~$\vec{\nabla}\times(\vec{\nabla}\times\phi)=0$~~ $\vec{\nabla}\times(\vec{\nabla}\phi)=0$

Sol:

$$\vec{\nabla}\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\vec{\nabla}\times(\vec{\nabla}\phi) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\phi}{\partial x} & \frac{\partial\phi}{\partial y} & \frac{\partial\phi}{\partial z} \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial^2\phi}{\partial y\partial z} - \frac{\partial^2\phi}{\partial z\partial y}\right) + \hat{j}\left(\frac{\partial^2\phi}{\partial z\partial y} - \frac{\partial^2\phi}{\partial x\partial z}\right)$$
$$+ \hat{k}\left(\frac{\partial^2\phi}{\partial x\partial y} - \frac{\partial^2\phi}{\partial y\partial x}\right) = 0$$

[$\because$ ~~ϕ ∂ Proved Necessary~~ We assume $\vec{A}$ has ~~continuous 2nd~~ continuous 2nd order partial derivatives]

② $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$ where $\vec{A}$ is a differentiable vector field.

$\cancel{\nabla\ \vec{\nabla}\times\vec{A}} =$

Sol: Let $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) + \hat{j}\left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right) + \hat{k}\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

$$= \frac{\partial^2 A_3}{\partial x\partial y} - \frac{\partial A_2}{\partial x\partial z} + \frac{\partial^2 A_1}{\partial y\partial z} - \frac{\partial^2 A_3}{\partial y\partial x} + \frac{\partial^2 A_2}{\partial z\partial x} - \frac{\partial^2 A_1}{\partial z\partial y}$$

$= 0$ [We assume $\vec{A}$ has continuous 2nd order partial derivative

③ $\cancel{\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) =}$ If $\vec{A}$ is a differentiable vector field then

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$ where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Sol: [H.W]

$\vec{\nabla}.\vec{\nabla} = \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is called Laplacian operator.

If ϕ and ψ are two differentiable scalar fields then.

~~$\vec{\nabla}.(\phi\vec{\nabla}\phi$~~ $\vec{\nabla}.(\phi\vec{\nabla}\psi - \psi\vec{\nabla}\phi) = \phi\nabla^2\psi - \psi\nabla^2\phi$

Sol:

$$\vec{\nabla}.(\phi\vec{\nabla}\psi) = \vec{\nabla}\phi.\vec{\nabla}\psi + \phi\nabla^2\psi$$

$$\vec{\nabla}.(\psi\vec{\nabla}\phi) = \vec{\nabla}\psi.\vec{\nabla}\phi + \psi\nabla^2\phi$$

$$\therefore \vec{\nabla}.(\phi\vec{\nabla}\psi - \psi\vec{\nabla}\phi) = (\phi\nabla^2\psi - \psi\nabla^2\phi).$$